Deriving the Sigmoid Derivative (Step by Step)

The sigmoid has a remarkably elegant derivative. Let's derive it from scratch:

Step 1: Write sigmoid as a power:

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Step 2: Apply the chain rule:

$$\frac{d\sigma}{dz} = -(1 + e^{-z})^{-2} \cdot \frac{d}{dz}(1 + e^{-z}) = -(1 + e^{-z})^{-2} \cdot (-e^{-z})$$

Step 3: Simplify:

$$\frac{d\sigma}{dz} = \frac{e^{-z}}{(1 + e^{-z})^2} = \frac{1}{1 + e^{-z}} \cdot \frac{e^{-z}}{1 + e^{-z}}$$

Step 4: Recognize the first fraction is σ(z), and the second is 1 − σ(z):

$$\frac{d\sigma}{dz} = \sigma(z) \cdot (1 - \sigma(z))$$